Characterization of relativistic electron energy spectra from CRRES observations

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Outline

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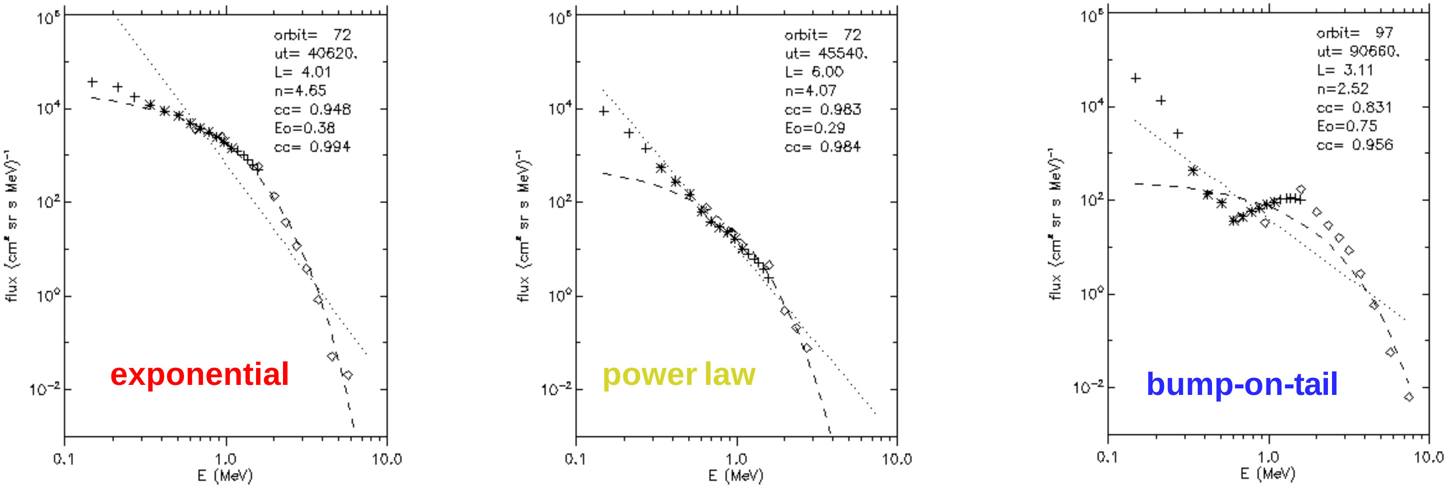
Introduction

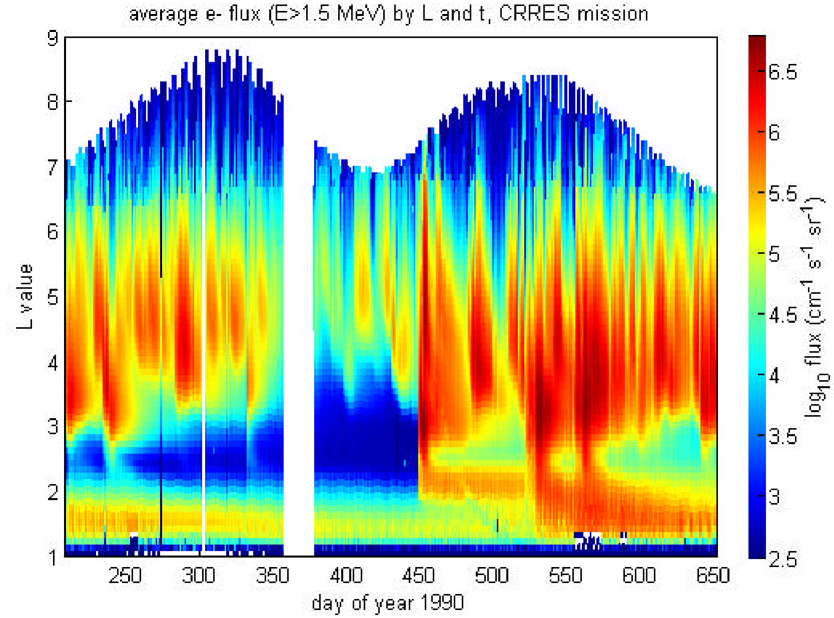
- The energy spectra of radiation belt electrons take a variety of shapes—exponential, power law, bimodal, “bump-on-tail”
  - Much variation with time and location
  - Sharp contrast with radiation belt protons



- Characterizing electron energy spectra is important for analyses such as spectral inversion of observations, cross-calibration between instruments
- Spectral variability is an aspect of radiation belt dynamics

Observations

- **CRRES:**
  - **Operational July 1990-Oct 1991, orbit 323 x 33790 km, 18? incl.**
- **Instruments used:**
  - **MEA: magnetic energy analyzer, 17 differential channels, 153 keV-1.58 MeV**
  - **HEEF: solid state particle telescope, 11 differential channels, 650 keV-8 MeV**
- **Total of 495,000 observations from L=2.5 to L=7-8.8 (one minute averages)**
- **All available observations were analyzed with two independent methods: data clustering and curve fitting**
  - **MEA and HEEF both provide pitch-angle resolved data, but omni-directional averages were used in this study**



Data Clustering

- **K-Means Data Clustering**
  - **Non-parametric method of grouping spectra based on distance metric**

*** Start with random cluster vectors ("centers")**

$$\vec{\mathbf{b}} = \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{pmatrix}$$

C_i is flux in i^{th} energy channel

Measurement vector

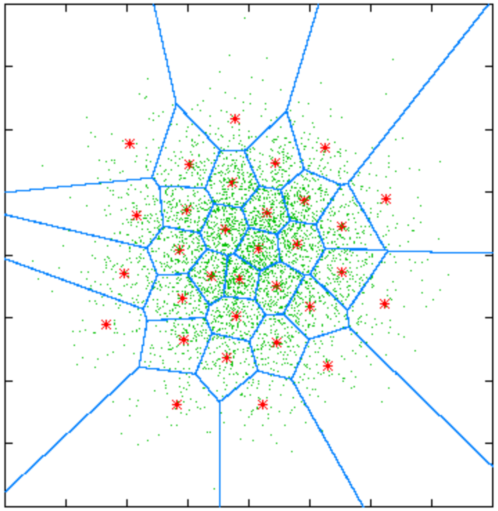
*** Assign each measurement to nearest cluster center**

$$d\left(\vec{\mathbf{b}}_1, \vec{\mathbf{V}}_k\right) = \left(\sum_{i=1}^{N} \left(b_{1i} - V_{ki}\right)^2 \right)^{1/2}$$

V_k is k^{th} cluster

*** Recompute centers as average of member measurement vectors; iterate**

$$\vec{\mathbf{V}}_k = \sum_{i \in \{\text{kth cluster}\}} \vec{\mathbf{b}}_i \;/(\text{Number of Elements in Cluster})$$



http://www.data-compression.com/vq.html

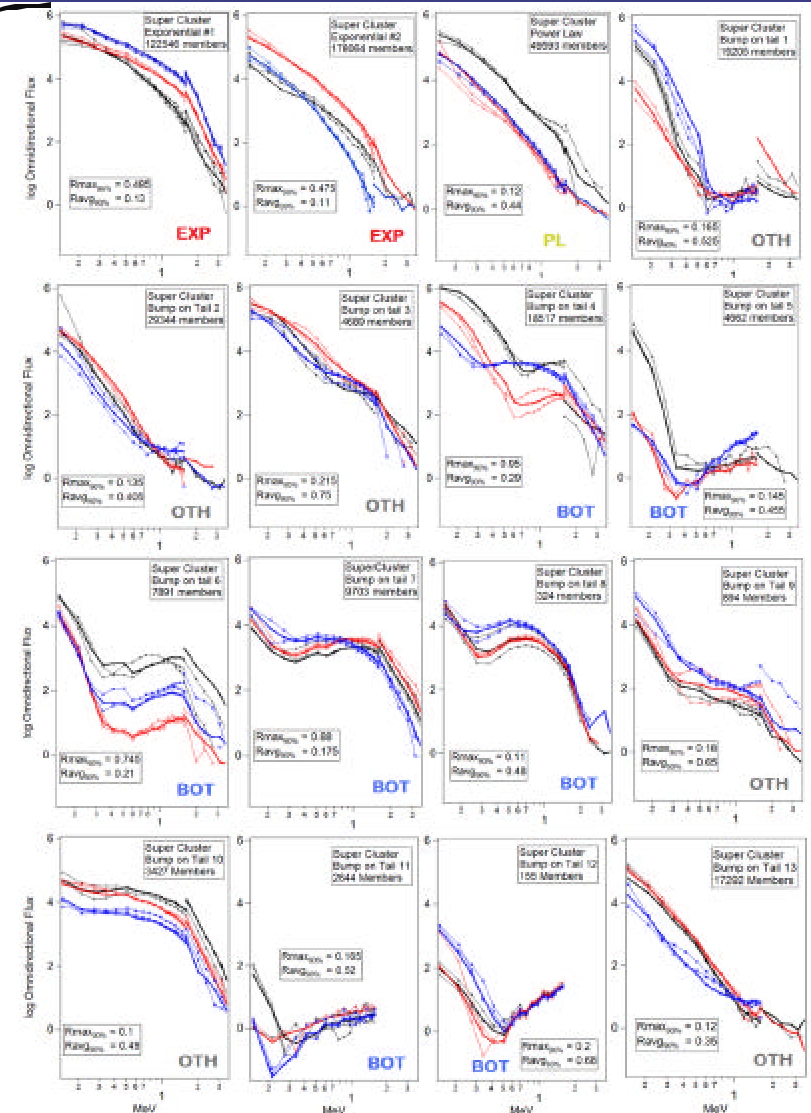
  - **Issues: Number of clusters, normalization, missing data, sub-optimal clustering**
    - **Missing data—restrict to energy channels/measurements with complete data**
    - **Number of clusters, normalization, & sub-optimal clustering – Use residuals to recluster exhaustively:**

$$\mathrm{R}_{\max}\left(\vec{\mathbf{b}}_1, \vec{\mathbf{V}}_k\right) = \max\left(\left|b_{1i} - V_{ki}\right|\right) \qquad \mathrm{R}_{\text{avg}}\left(\vec{\mathbf{b}}_1, \vec{\mathbf{V}}_k\right) = \frac{1}{N} \sum_{i=1}^{N} \left|b_{1i} - V_{ki}\right|$$

Jain et al. (1999), *ACM Comp. Surv.*, 31(3):264+; Lindstrom et al. (2009), *AIAA J*, 47:2379.

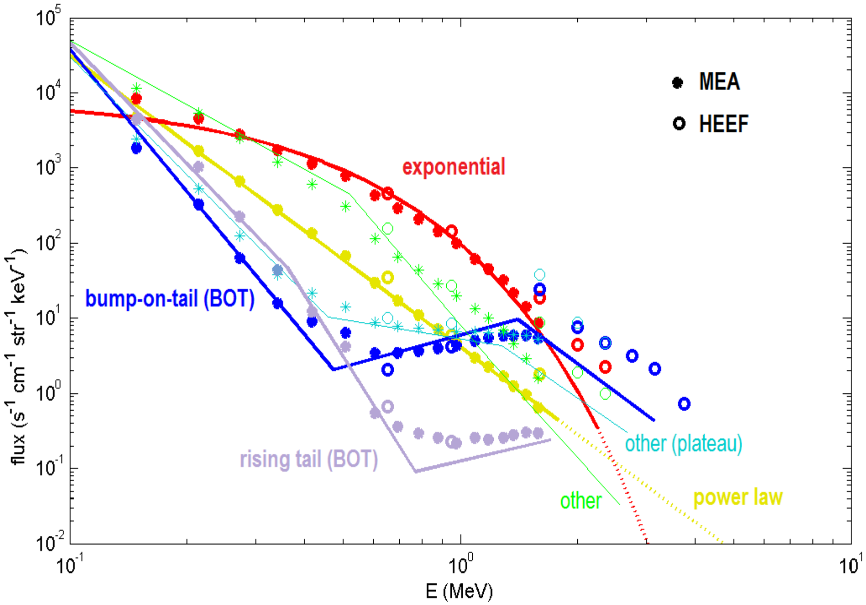
Clustering method

- For clustering, observations were binned in 0.5-L bins (except one bin for L=7-9)
  - Total: 485,771 observations (L>2.5)
- K-means clustering was applied to MEA spectra (log values) for each L-bin separately
  - MEA data nearly complete
  - Result: 1532 subclusters (typically ~100 per L-bin)
- For each subcluster, the average MEA-HEEF spectra was visually classified into superclusters based on shape
  - Hand-picking was done in order to sort on shape without bias from magnitude
  - Result: 16 superclusters
- These 16 superclusters may be classified as exponential (2), power law (1), and everything else (13)
  - "Everything else" includes cases where cluster slope is not constant or monotonically decreasing over MEA range (<1.6 MeV)
  - At the right these are subdivided into bump-on-tail = BOT (7), which have local minima, and other unusual (6)



Curve fitting method

- Each MEA-HEEF spectrum was fit with three curves
  - Exponential $J = J_0 e^{-E/Eo}$
  - Power law $J = b E^{-n}$
  - 3-segment broken power law (BPL)
- Sum of squared errors (SSEs) compared for all three fits:
  - If SSE_{EXP} or $SSE_{PL} < 3 * SSE_{BPL}$, classify as exponential or power law (whichever is better)
  - This addresses the bias from more fit parameters with BPL (6 vs. 2)
- Remaining spectra are BPL—these are divided into classes based on fit parameters:
  - Local minima → bump-on-tail (BOT)
  - Everything else → other (OTH)
  - Two subclasses of each are shown at right
- Issues for both methods:
  - HEEF data availability is limited, giving bias toward observations with higher fluxes
  - Noise floor in both instruments may influence shape



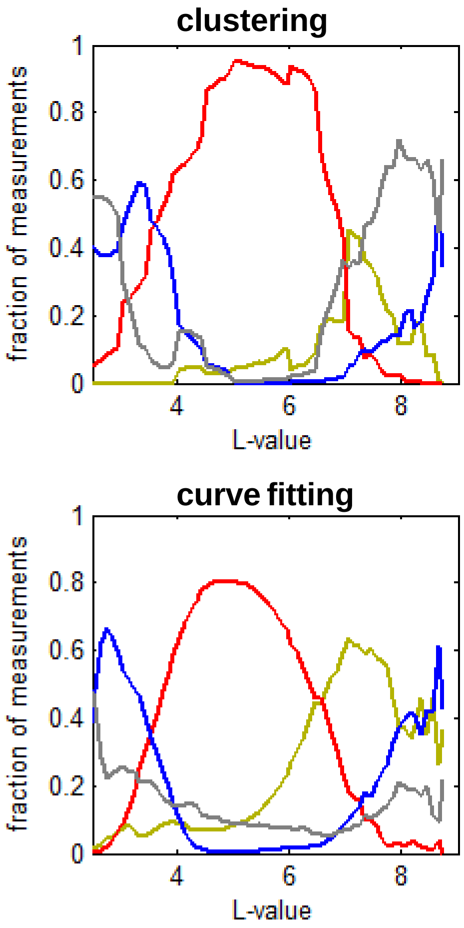
- Plot shows the average results for curve fit groups
  - Solid lines = average of fit parameters
  - Markers = log average of MEA (*) and HEEF (o) measurements for group members
  - HEEF results are shown only where data exists for at least 1/3 of group members

Comparison of the two methods

- **Results are two independent methods classifying electron energy spectra**

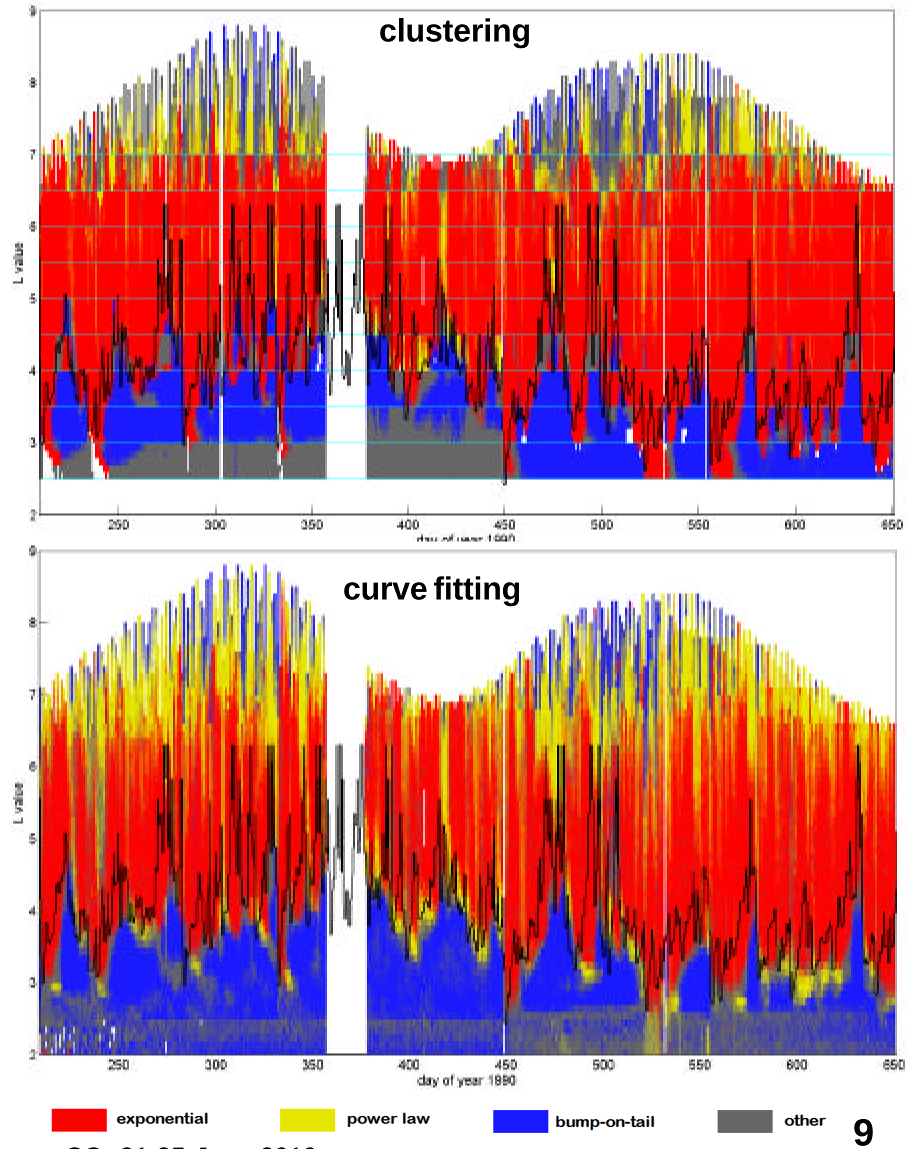
	n	EXP	PL	BOT	OTH
clustering	467317	64.3%	10.5%	9.4%	15.8%
curve fitting	494605	49.2%	28.8%	11.2%	10.8%

- **Comparison of 466,472 spectra classified by both methods (4 classes):**
  - **64.4% same class**
  - **20.0% curve fit as PL but not by clustering**
  - **7.0% different BOT/other breakdown**
  - **8.6% other differences**
  - **Differences are often linked to whether or not HEEF data is used**
- **Similar distribution in L value**
  - **Bump-on-tail at L<3.5-4**
  - **Exponential at L=4-6.5**
  - **Transition to power law and other forms at higher L**



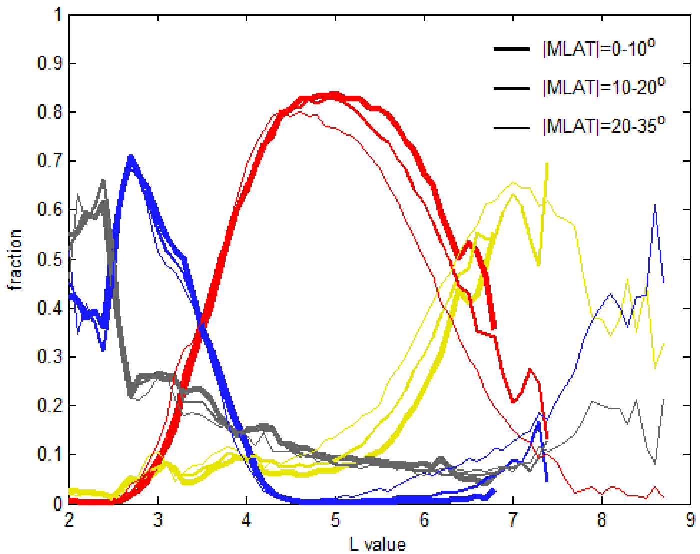
Spectral classes—L and t

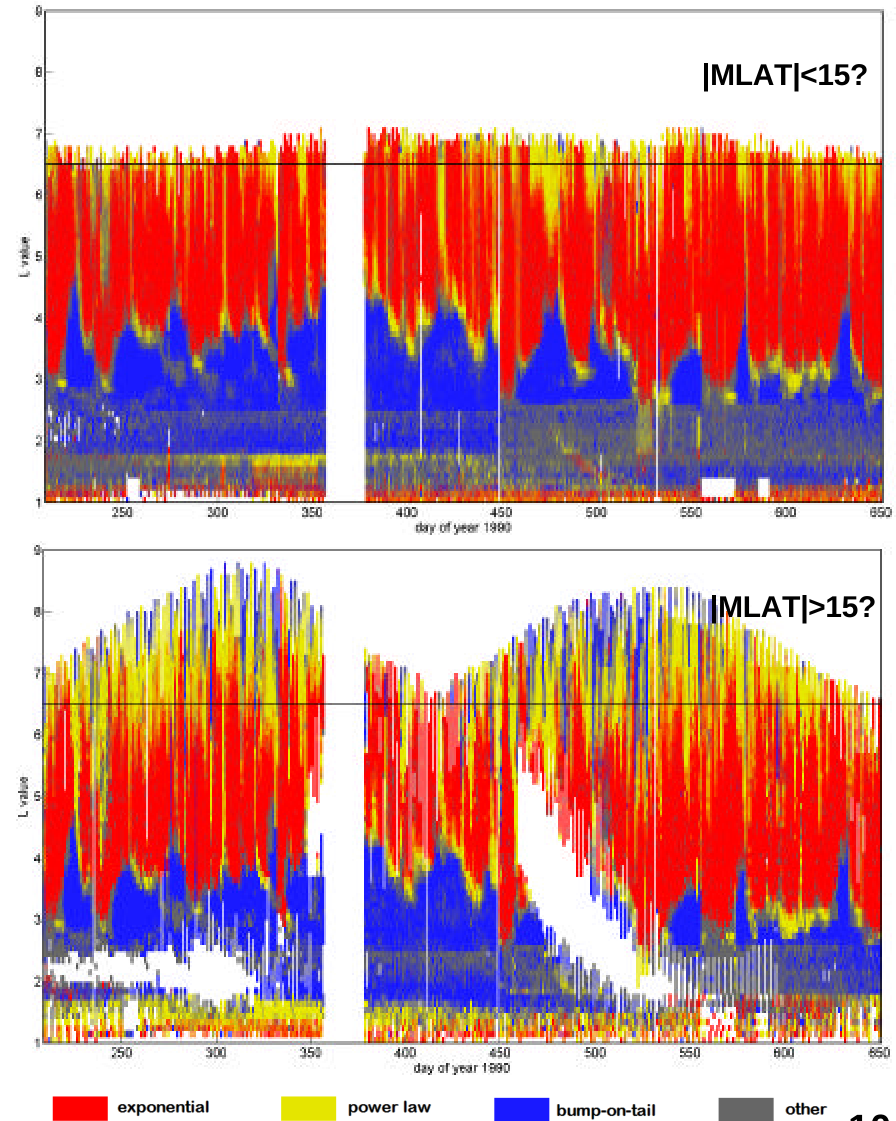
- Both clustering and curve fitting results show similar dynamics in the distribution of spectral types over L and time
  - black line = O'Brien-Moldwin model plasmapause
- Exponential spectra most common in outer belt
- Power law spectra most common at outskirts of outer belt
- Bump-on-tail most common in slot region
- Frequency of other shapes at L<2.5 partly reflects the issue of proton contamination in MEA
- Transition between BOT and exponential correlates with plasmapause location



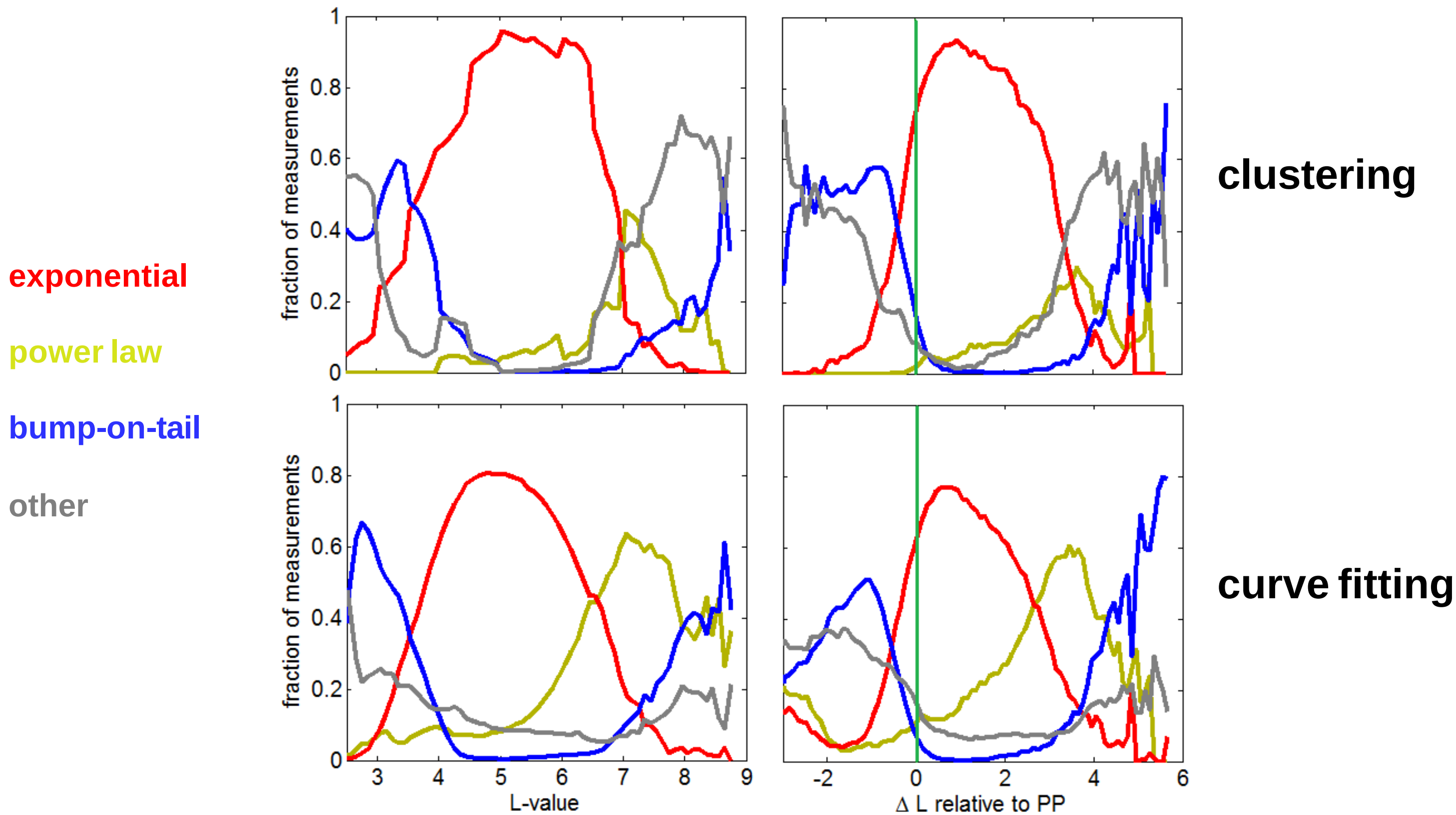
Spectral dependence on MLAT

- **The location of the transition from exponential to power law distributions at high L values may be an artifact of CRRES sampling**
  - **CRRES only sampled L>~6.5 for MLAT>15°**
- **Exponential distribution extends to larger L at lower MLAT**
  - **Relates to pitch-angle dependence of spectral form, which we have not examined yet**
  - **Similar MLAT-dependence not observed at low L values**



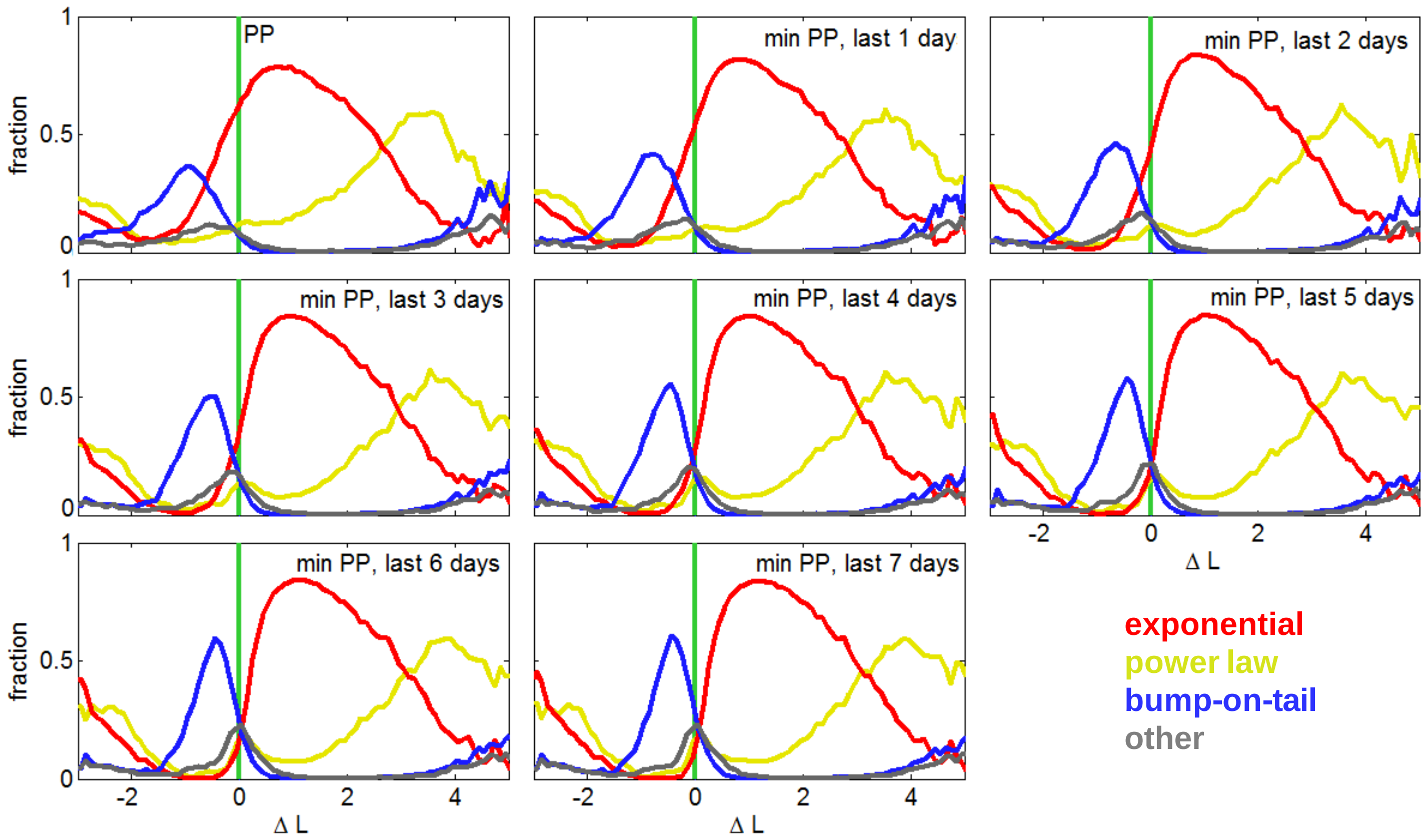


Spectral dependence on L



- **Spectral distribution relative to the plasmapause location shows a sharper low-L cutoff for exponential shapes (than distribution vs. L)**
  - **Plasmapause location from O'Brien-Moldwin model**

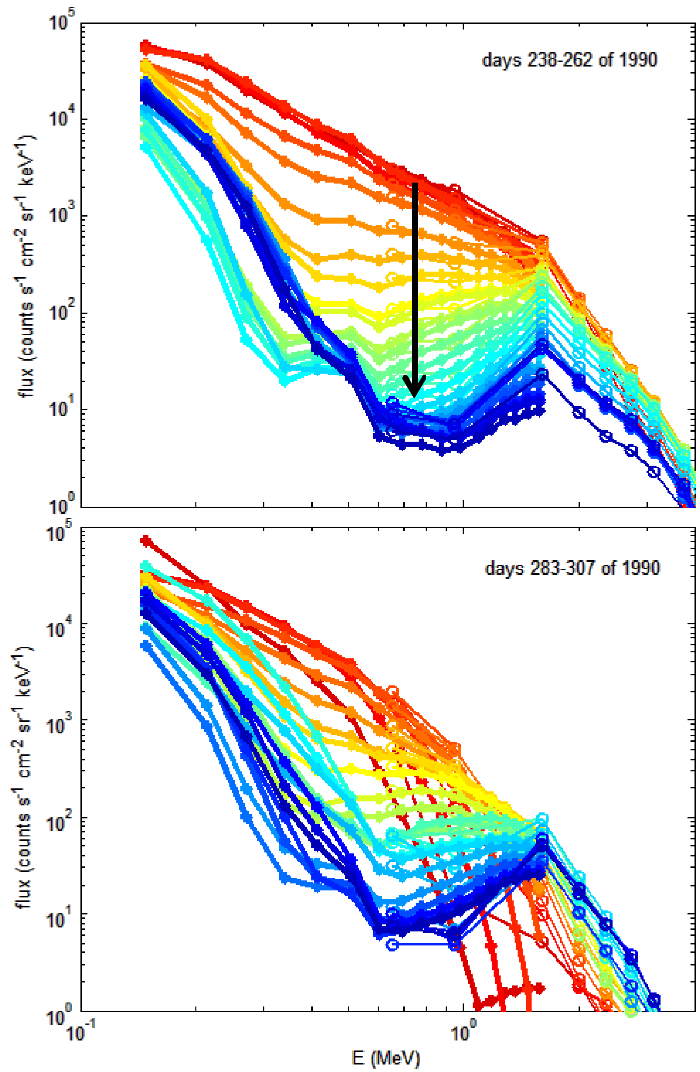
Spectral classes and the plasmapause



- **Division between BOT and exponential is more strongly linked to delayed plasmapause location**
  - **Good fit with 5-day minimum plasmapause location**
  - **Power law and other shapes peak at minimum plasmapause → transitional spectra**

Spectral classes—bump on tail

- **BOT distributions are observed to develop in the slot region following storms**
  - **Plots show development of BOT at L=3.2 following two storms (from red to blue, curves at one-day intervals)**
- **Characteristic BOT minimum at ~600 keV, maximum at ~1.5 MeV**
  - **Possible second minimum at ~350 keV**
  - **The crossover from MEA to HEEF makes it hard to precisely define the maximum location**
  - **However, similar max/min locations were noted in Ogo 5 data by West et al. (1981, *JGR*, 86:2111)**
- **Development of BOT results from energy-dependent losses due to wave-particle interactions with whistler hiss within the plasmasphere (Imhof et al., 1983, *JGR* 88:8103; Meredith et al., 2007, *JGR* 112:A08214.**



Conclusions

- **Electron energy spectral types are a function of location and are dynamic over time**
  - Exponential in the main outer belt, power law at higher L values, and BOT in the slot region
  - Transition from exponential to power law spectra takes place at higher L values for lower MLAT
- **The boundary between BOT and exponential spectra strongly correlates with plasmapause location, reflecting the role of plasmaspheric hiss in BOT development**
  - Good match to a 5-day minimum of the O'Brien-Moldwin plasmapause location
  - Modeling slot region BOT with a broken power law generally yields a minima at 350-600 keV and a maxima at 1.5-2 MeV
  - Such BOT is observed to develop following storms, the result of energy-dependent losses to wave-particle interactions with plasmaspheric hiss
- **A large fraction of cases (~60-90% at L=4-8) are well represented by simple exponential or power-law curves, but…**
- **The other cases are not**
  - The nature of the BOT spectral shape complicates curve-fitting, spectral inversion, etc.
  - Various bi-modal distributions have been successfully used in the literature at some locations (e.g. geosynchronous)—this is not feasible for inversion of data with limited numbers of channels, though
- **Topics/issues for future work:**
  - examining pitch angle dependence - revisit after further cross-calibration of HEEF, MEA
  - fitting other types of curves - using principle components analysis